Hands-On Session

C2: Stochastic thermodynamics of heat and information engines

XIV GEFENOL-DIFENSC Summer School on Statistical Physics of Complex Systems

Alcaláde Henares, July 6-17, 2026

I. Information-Energy Exchange under symmetric protocol

a) $\gamma\dot{x} = -V'(x) + \sqrt{2k_BT\gamma}\zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = \delta(t-t')$

Derive equilibrium probability distribution of position $[P_{eq}(x)$ of overdamped Brownian Particle in harmonic potential. The potential is $V(x) = \frac{\kappa}{2}x^2$; where κ is the stiffness of the trap. Following that calculate average potential energy $\langle V\rangle$ of the system under consideration?

b) Construct a Brownian Information Engine following error-free symmetric protocol (shown on the last page). Evaluate the potential change after feedback.

c) What will be the free energy change $\langle\Delta F\rangle$ associated with the cycle. Calculate

1. The average work extracted $-\langle W\rangle$
2. The average information contents $\langle I\rangle$ and $\langle I_u\rangle$

d) Comment on the following relations

1. Average potential energy $\langle V\rangle$ and average work extraction $-\langle W\rangle$
2. Average work extraction $-\langle W\rangle$ and information content $\langle I\rangle$ and $\langle I_u\rangle$

II. Influence of potential trapping in Information-Energy

a) $\gamma\dot{x} = -V'(x) + \sqrt{2k_BT\gamma}\zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = \delta(t-t')$

Derive equilibrium probability distribution of position $[P_{eq}(x)$ of overdamped Brownian Particle in harmonic potential. The potential is $V(x) = a|x|^n$, with $n > 0$; where a is the strength of the trap. Comment on the $V(x)$ and $P_{eq}(x)$ for the following three cases: $1.\, n = 1;\quad 2.\, n > 1;\quad 3.\, n < 1$?

b) Following the result in (a) evaluate the average potential energy $\langle V \rangle$

c) Construct a Brownian Information Engine following an error-free symmetric protocol (shown at end). What's the potential change after the feedback?

d) What is the free energy change $\langle \Delta F \rangle$ associated with the cycle?

Calculate ,

1. average work extraction $-\langle W \rangle$

   Comment on the work output from the following three cases:

   $n = 1;\quad n > 1;\quad n < 1$

2. the average information content $\langle I \rangle$ *and* $\langle I_u \rangle$

e) Comment on the following relations

1. Average potential energy $\langle V \rangle$ and average work extraction $-\langle W \rangle$
2. Average work extraction $-\langle W \rangle$ and information content $\langle I \rangle$ *and* $\langle I_u \rangle$

Schematic representation of Symmetric Protocol

